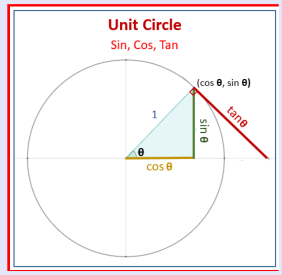
Trigonometry

Lecture 26



Feb 19-8:47 AM

$\sin 2x = 2\sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = 2\cos^2 x - 1$

$\cos 2x = 1 - 2\sin^2 x$

$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

Suppose

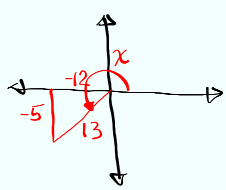
$\sin x = \frac{-5}{13}$

x is is in QIII

Find

$\sin 2x$, $\cos 2x$, and

$\tan 2x$.



$\sin 2x = 2\sin x \cos x$

$= 2 \cdot \frac{-5}{13} \cdot \frac{-12}{13} = \frac{120}{169}$

$\cos 2x = \cos^2 x - \sin^2 x = \left(\frac{-12}{13}\right)^2 - \left(\frac{-5}{13}\right)^2 = \frac{144}{169} - \frac{25}{169} = \boxed{\frac{119}{169}}$

$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x} = \dfrac{2 \cdot \frac{5}{12}}{1 - \left(\frac{5}{12}\right)^2} = \dfrac{\frac{10}{12}}{1 - \frac{25}{144}} = \dfrac{144 \cdot \frac{10}{12}}{144 - 25}$

LCD = 144 $= \boxed{\frac{120}{119}}$

Oct 14-10:28 AM

Verify $\dfrac{\sin 3x}{\sin x \cos x} = 4\cos x - \sec x$ ✓

$3x = x + 2x$

$\sin 3x = \sin(x + 2x)$

$= \sin x \cos 2x + \cos x \sin 2x$

$= \sin x\left[2\cos^2 x - 1\right] + \cos x(2\sin x \cos x)$

$= 2\sin x \cos^2 x - \sin x + 2\sin x \cos^2 x$

$= 4\sin x \cos^2 x - \sin x$

$$\frac{\sin 3x}{\sin x \cos x} = \frac{4\sin x \cos^2 x - \sin x}{\sin x \cos x}$$

$$= \frac{4\cancel{\sin x}\cancel{\cos^2} x}{\cancel{\sin x}\cancel{\cos x}} - \frac{\cancel{\sin x}}{\cancel{\sin x}\cos x}$$

$$= \boxed{4\cos x - \sec x}$$

Oct 14-10:36 AM

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$$

$$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$$

Determine + or − based on the quadrant of $\frac{x}{2}$.

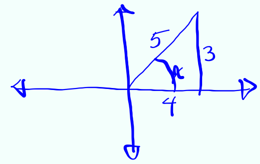
$$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x} = \frac{\sin x}{1+\cos x}$$

Oct 14-10:42 AM

Given $\sin x = \frac{3}{5}$, x is QI $0^\circ < x < 90^\circ$

Find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$, $\tan\frac{x}{2}$ $\quad 0^\circ < \frac{x}{2} < 45^\circ$

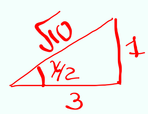
$$\sin\frac{x}{2} = +\sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1-\frac{4}{5}}{2}}$$



$$= \sqrt{\frac{5-4}{10}} = \sqrt{\frac{1}{10}} = \frac{1}{\sqrt{10}} = \boxed{\frac{\sqrt{10}}{10}}$$

$$\cos\frac{x}{2} = +\sqrt{\frac{1+\cos x}{2}} = \sqrt{\frac{1+\frac{4}{5}}{2}} = \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}} = \boxed{\frac{3\sqrt{10}}{10}}$$

$$\tan\frac{x}{2} = \frac{\sin x}{1+\cos x} = \frac{\frac{3}{5}}{1+\frac{4}{5}} = \frac{3}{9} = \boxed{\frac{1}{3}}$$



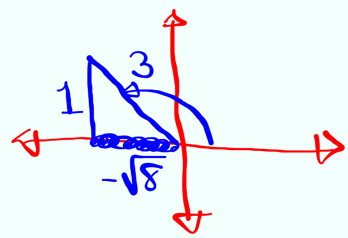
$$\sin\frac{x}{2} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$$

$$\cos\frac{x}{2} = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$$

Oct 14-10:45 AM

$\csc x = 3$, $90^\circ < x < 180^\circ$

$\rightarrow \sin x = \frac{1}{3}$



Find $\cos\frac{x}{2}$

$$45^\circ < \frac{x}{2} < 90^\circ$$

QI

$$\cos\frac{x}{2} > 0$$

$$\cos\frac{x}{2} = +\sqrt{\frac{1+\cos x}{2}} = \sqrt{\frac{1-\frac{\sqrt{8}}{3}}{2}} = \boxed{\sqrt{\frac{3-\sqrt{8}}{6}}}$$

3

Oct 14-10:54 AM

Product - to - Sum

$\sin x \cos y = \frac{1}{2}[\sin(x+y) + \sin(x-y)]$

$\cos x \sin y = \frac{1}{2}[\sin(x+y) - \sin(x-y)]$

$\cos x \cos y = \frac{1}{2}[\cos(x+y) + \cos(x-y)]$

$\sin x \sin y = \frac{1}{2}[\cos(x-y) - \cos(x+y)]$

$\sin 3x \cos 5x = \frac{1}{2}[\sin(3x+5x) + \sin(3x-5x)]$

$= \frac{1}{2}[\sin 8x + \underbrace{\sin(-2x)}]$

$= \boxed{\frac{1}{2}[\sin 8x - \sin 2x]}$

$\sin 3x \sin 5x = \frac{1}{2}[\cos(3x-5x) - \cos(3x+5x)]$

$= \frac{1}{2}[\underbrace{\cos(-2x)} - \cos 8x]$

$= \frac{1}{2}[\cos 2x - \cos 8x]$

Oct 14-10:59 AM

Write $\cos x \cos 5x$ as a Sum.

$\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

$\cos x \cos 5x = \frac{1}{2}[\cos(x+5x) + \cos(x-5x)]$

$= \frac{1}{2}[\cos 6x + \underbrace{\cos(-4x)}]$

$= \frac{1}{2}[\cos 6x + \cos 4x]$

Oct 14-11:07 AM

Sum - to - Product Formula

$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$

$\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$

$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$

$\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$

$\sin 3x + \sin x = 2\sin\frac{3x+x}{2}\cos\frac{3x-x}{2}$

$= \boxed{2\sin 2x\cos x}$

$\cos 3x - \cos 7x = -2\sin\frac{3x+7x}{2}\sin\frac{3x-7x}{2}$

$= -2\sin 5x \underbrace{\sin(-2x)}_{-\sin 2x}$

$= \boxed{2\sin 5x\sin 2x}$

Oct 14-11:11 AM

Write $\cos 10x + \cos 2x$ as a product

$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$

$\cos 10x + \cos 2x = 2\cos\frac{10x+2x}{2}\cos\frac{10x-2x}{2}$

$= 2\cos 6x\cdot\cos 4x$

find the exact value of

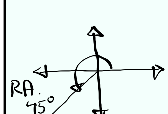
$\cos 255° - \cos 195° =$

$\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$

$= -2\sin\frac{255°+195°}{2}\sin\frac{255°-195°}{2}$

$= -2\cdot\sin\frac{450°}{2}\sin\frac{60°}{2}$

$= -2\sin 225°\sin 30°$



$= \cancel{-2}\cdot -\sin 45°\cdot\frac{1}{\cancel{2}}$

$= \sin 45° = \boxed{\frac{\sqrt{2}}{2}}$

Oct 14-11:19 AM

Find exact value of

$$\cos\frac{\pi}{12}+\cos\frac{5\pi}{12}=2\cos\frac{\frac{\pi}{12}+\frac{5\pi}{12}}{2}\cos\frac{\frac{\pi}{12}-\frac{5\pi}{12}}{2}$$

$$\cos x+\cos y=2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$

$\frac{\pi}{12}=15^\circ$, $\frac{5\pi}{12}=75^\circ$

$$=2\cos\frac{15^\circ+75^\circ}{2}\cdot\cos\frac{15^\circ-75^\circ}{2}$$

$$=2\cdot\cos 45^\circ\cdot\cos(-30^\circ)$$

$$=2\cdot\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}=\boxed{\frac{\sqrt{6}}{2}}$$

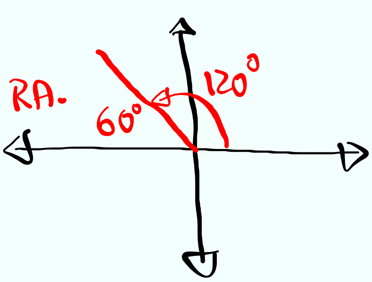
Oct 14-11:28 AM

Show $\sin 130^\circ-\sin 110^\circ=-\sin 10^\circ$

Hint:

$$\sin x-\sin y=2\cos\frac{x+y}{2}\cdot\sin\frac{x-y}{2}$$

$$\sin 130^\circ-\sin 110^\circ=2\cos\frac{130^\circ+110^\circ}{2}\sin\frac{130^\circ-110^\circ}{2}$$



$$=2\cdot\cos 120^\circ\cdot\sin 10^\circ$$

$$=2\cdot-\cos 60^\circ\cdot\sin 10^\circ$$

$$=-\cancel{2}\cdot\frac{1}{\cancel{2}}\cdot\sin 10^\circ$$

$$=-\sin 10^\circ$$

Oct 14-11:33 AM

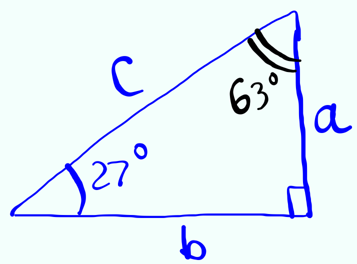
Simplify

$$\cos 87^\circ + \cos 33^\circ = 2\cos\frac{87^\circ+33^\circ}{2}\cos\frac{87^\circ-33^\circ}{2}$$

$$= 2\cos 60^\circ \cdot \cos 27^\circ$$

Hint:

$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$



$$= 2 \cdot \frac{1}{2}\cos 27^\circ$$

$$= \cos 27^\circ = \frac{b}{c}$$

$$= \sin 63^\circ$$

Oct 14-11:37 AM